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Review of Models for Thermal Contact Conductance of Metals

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Thermal contact conductance is a topic of great relevance to such applications as electronics packaging, satellite thermal control, nuclear reactor cooling, aerodynamic heating of supersonic aircraft and missiles, and turbine and internal combustion engine design. A fundamental problem in this discipline concerns the contact of metals in a vacuum environment for which gap and radiative conductance are negligible and only contact (solid spot) conductance is appreciable. A number of conductance models for metals are compiled and compared to experimental data from the literature. Theoretical models have been developed that accurately predict contact conductance for the two bounding cases of flat, rough surfaces and nonflat (spherical), smooth surfaces. However, these do not agree with most results for arbitrarily nonflat, rough surfaces, those usually obtained from common manufacturing processes. Empirical and semiempirical correlations, although many are developed for nonflat, rough surfaces, also suffer from limited applicability. The few theoretical models for nonflat (spherical), rough surfaces are computationally intensive and are not readily applied to design.

Nomenclature

A = area, m^2
a_L = radius of macrocontact region, m
a_S = radius of microcontact, m
B = bulk coefficient of thermal expansion, K^{-1}
b_L = component radius, m
b_S = radius of elemental heat flux channel
$C_{1,2,3}$ = constant or function in literature
c_1 = microhardness correlation coefficient, N/m^2
D = fractal dimension of surface profile
D_V = diagonal length of Vickers indentation, μm
E = Young's modulus of elasticity, N/m^2
E' = Hertz[36] elastic modulus, N/m^2, $E' = [(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2]^{-1}$
H = hardness, N/m^2
h = thermal conductance, W/m^2K
$I_{0,0}$, $I_{1,1}$ = integrals derived by Whitehouse and Archard,[2] recast by Sridhar and Yovanovich[34]
I_1 = integral derived by Greenwood and Williamson,[1] recast by Sridhar and Yovanovich[34]
J_0, J_1 = Bessel functions of the first kind
$K(\xi)$ = complete elliptic integral of modulus ξ
k = harmonic mean thermal conductivity, W/mK, $k = 2k_1k_2/(k_1 + k_2)$
L = load, N
m = combined mean absolute profile slope, m/m, $m = (m_1^2 + m_2^2)^{1/2}$
N = total number of microcontacts
P = pressure, N/m^2
Q = heat rate, W
q = heat flux, W/m^2
R = thermal resistance, m^2K/W
R'' = thermal resistance not normalized for area, K/W
r = radial displacement along surface from center of axisymmetric contact, m
S = material strength, N/m^2
T = absolute temperature, K
TIR = combined flatness deviation (Total Included Reading), m, $TIR = TIR_1 + TIR_2$
t = thickness, m
u = mean plane separation, m
V = volume of voids between contacting surfaces, m^3
Var = statistical variance, units vary
VHN = Vickers hardness number, kg/mm^2
w = deformation due to load, m
x = displacement, Cartesian coordinates, m
z = height above mean plane of surface, m
α = bandwidth of surface profile measurements, $\alpha = 4\sigma^2 a_L \,\mathrm{Var}(d^2\sigma/dx^2)/(\pi^2 m^2)$
β = radius of curvature of asperity, m
γ = load redistribution parameter
Δx = sampling interval for roughness and profile slope, m
δ = combined crown drops of surfaces, m, $\delta = \delta_1 + \delta_2$
ε = a_s/b_s or a_L/b_L
ζ_n = nth root of Bessel function $J_1(\zeta_n)$
η = number of asperities (peaks) per unit area, $1/m^2$
θ = angle between banks of grooves and mean plane, rad
κ = function of elliptic integral $K(\xi)$, [$\kappa = (2/\pi)K(\xi)$ for $\xi \leq 1$; $\kappa = (2/\pi)K(1/\xi)$ for $\xi > 1$]
λ = effective fluid gap, m, $\lambda = f(\sigma_{1,CLA} + \sigma_{2,CLA})$
μ = micro = 10^{-6}
ν = Poisson ratio
ξ = modulus of elliptic integral K
ρ = combined radius of curvature, m, $\rho = (1/\rho_1 + 1/\rho_2)^{-1}$
σ = combined root-mean-square (rms) roughness, m, $\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2}$

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ϕ	=	probability density function of peak heights
χ	=	elastic conformity modulus derived by Clausing and Chao[3]
ψ	=	thermal constriction alleviation factor
ω	=	transverse wavelength of grooves, m

Subscripts

app	=	apparent
av	=	average, mean at interface
B	=	Brinell macrohardness
CLA	=	Center Line Average roughness or profile slope
c	=	contact microhardness (as in H_c) or contact conductance (as in h_c)
el	=	elastic deformation
ep	=	elasto–plastic deformation
f	=	fluid property
film	=	film
g	=	gas property (as in k_g) or gap conductance (as in h_g)
Hz	=	Hertz[36] (pertaining to a smooth sphere)
harm	=	harmonic mean
i	=	index of summation or individual asperity
j	=	index of summation
junc	=	junction conductance, as in h_{junc} (sum of contact, gap, and radiative conductances)
K	=	Knoop microhardness
L	=	large scale, macroscopic, or bulk, as in b_L, h_L, or H_L
M	=	Meyer macrohardness
max	=	maximum
melt	=	melting temperature
min	=	minimum
n	=	index of summation
oxide	=	pertaining to oxidized surface layer
P=0	=	no load value, i.e., prior to contact
pl	=	plastic deformation
RMS	=	root-mean-square
rad	=	radiative conductance
real	=	real
rgh	=	rougher surface
S	=	small scale, microscopic
slope	=	pertaining to profile slope
smth	=	smoother surface
solid	=	solid
U	=	ultimate strength, as in S_U
V	=	Vickers microhardness
Y	=	yield strength, as in S_Y
0	=	at axis or center of contact
1	=	specimen or surface 1
2	=	specimen or surface 2

Superscripts

$'$	=	effective
*	=	dimensionless

I. Literature Review

A NUMBER of reviews of various aspects of thermal contact conductance are available in the literature. More recent ones include Madhusudana and Fletcher,[4,5] Snaith et al.,[6] Yovanovich,[7] and Fletcher.[8,9] Those studies of thermal contact conductance of metals are reviewed next.

A. Thermal Constriction Resistance

Roess[10] derived the microcontact resistance of a quasi-isothermal, circular contact spot:

$$R''_{c,S,i} = \frac{\psi(a_{S,i}/b_{S,i})}{2ka_{S,i}} \tag{1}$$

The constriction alleviation factor, $\psi(a_{S,i}/b_{S,i})$, accounts for interaction of adjacent microcontacts. It equals one when each surface is a half-space ($b_{S,i} \to \infty$), and is less than one for a finite heat flux channel of radius $b_{S,i}$. Yovanovich[11] generalized the solution to include uniform heat flux over the microcontact. Cooper et al.[12] derived a simple expression for $\psi(\varepsilon_S)$:

$$\psi(\varepsilon_S) = \psi(a_S/b_S) = (1 - \varepsilon_S)^{1.5} \tag{2}$$

The overall microcontact resistance is

$$R_{c,S} = \frac{1}{h_{c,S}} = \frac{\psi(a_S/b_S)}{2k\eta a_S} = \frac{(1 - \varepsilon_S)^{1.5}}{2k\eta a_S} \tag{3}$$

Roess'[10] expression is widely incorporated in models for thermal contact conductance, which involve determination of the ratio a_S/b_S on the basis of deformation models and surface profiles.

B. Empirical and Semiempirical Correlations for Nominally Flat, Rough Metals

A number of empirical and semiempirical correlations for the contact conductance of nominally flat (though actually arbitrarily nonflat), rough surfaces are listed in Table 1. The correlations by Cetinkale and Fishenden,[13] Fenech and Rohsenow,[14,15] Laming,[16] Tachibana,[17] and Veziroglu[18] include the contribution to joint conductance resulting from gap conductance of fluid in the interstices between microcontacts. The correlation by Veziroglu[18] includes the experimental results of the other four correlations listed earlier. For vacuum pressures below approximately 10^{-1} torr, the fluid conductivity k_f is approximately zero. All of these models use an empirically derived gap thickness $\boldsymbol{\lambda}$. The empirical correlation by Fletcher and Gyorog[19] is the only one in Table 1 that addresses both macrocontact and microcontact resistance.

The correlations of the Russian investigators, Shlykov and Ganin,[20] Shlykov,[21] Mal'kov,[22] Bochorishvili and Ganin,[23] and Popov,[24] assume the contact microhardness H_c equals three times the ultimate strength S_U. Some include an empirical correction factor C_1. Of the empirical correlations listed in Table 1, only Cetinkale and Fishenden[13] and Fletcher and Gyorog[19] explicitly considered flatness deviation, TIR, which is included in their correlations. Laming[16] modeled the contact of directionally wavy surfaces. The correlation by Fletcher and Gyorog[19] was based on 400 conductance measurements for aluminum, brass, magnesium, and stainless steel. Thomas and Probert[25] and O'Callaghan and Probert[26] both used approximately 340 results for aluminum and stainless steel. Although obtained for large bodies of data, these correlations do not exhibit general applicability.

C. Theoretical Models for Flat, Rough Metals

Theoretical models for the thermal contact conductance of conforming (e.g., flat), rough metals are listed in Table 2. In practice, conforming surfaces are nearly optically flat, which are herein defined to have a combined TIR of less than 2.0 μm.

Hegazy[27] demonstrated through experiments with four alloys (S.S. 304, nickel 200, Zr-2.5% Nb, and Zircaloy-4) that the contact microhardness H_c is significantly greater than the bulk hardness or macrohardness H_L, due to work hardening of metallic surfaces during machining. He also showed that H_c decreases with increasing indentor penetration until H_L is eventually obtained. He derived an empirical correlation to relate H_c of the softer surface to the depth of penetration of asperities on the harder surface:

$$\frac{H_c}{10^9} = \left(12.2 - \frac{3.54H_L}{10^9}\right)\left(\frac{\sigma \times 10^6}{m}\right)^{-0.26} \tag{4}$$

Table 1 Empirical and semiempirical correlations for thermal contact conductance of nominally flat (arbitrarily nonflat), rough metals

Reference	Correlation	Remarks
13 (semiempirical)	$$\frac{h_{c+g,S}\sigma}{km} = \frac{k_f\sigma}{\lambda km} + \frac{\sigma a_S}{m b_S^2 \tan^{-1}\left(\frac{b_S}{a_S}\sqrt{1-\frac{k_f}{h_{c+g,S}\lambda}}-1\right)}$$ $\lambda = 0.61(\sigma_{1,CLA} + \sigma_{2,CLA})$ $b_S = 0.0048(TIR_1 + TIR_2) \times [(P_{max}^{1/3} \times P^{2/3})/H_M]^{1/2}$; elastic $b_S = 0.0048(TIR_1 + TIR_2) \times (P/H_M)^{1/2}$; plastic $P_{max} \approx 10$ MPa for experimental data by Hegazy[27] in Figs. 3a and 3b	1) Contact and gap conductance 2) Flat, rough surfaces 3) Steel, brass, and aluminum with ground surfaces 4) Air, spindle oil, and glycerol in gaps
14, 15 (semiempirical)	$$\frac{h_{c+g,S}\sigma}{km} = \frac{\frac{k_f\sigma}{km}\left[\left(1-\frac{a_S^2}{b_S^2}\right)C_1 + 1.1\frac{a_S}{b_S}\psi\left(\frac{a_S}{b_S}\right)\left(\frac{1}{k_1}+\frac{1}{k_2}\right)\right] + 4.26\frac{a_S}{b_S}\sigma\sqrt{\eta}}{(\lambda_1+\lambda_2)\left(1-\frac{a_S^2}{b_S^2}\right)\left[1-\frac{k_f}{\lambda_1+\lambda_2}\left(\frac{\lambda_1}{k_1}+\frac{\lambda_2}{k_2}\right)\right]C_1}$$ $$C_1 = \frac{4.26\sqrt{\eta}\lambda_1 b_S}{k_1 a_S} + \frac{1}{k_1} + \frac{4.26\sqrt{\eta}\lambda_2 b_S}{k_2 a_S} + \frac{1}{k_2}$$ λ_1, λ_2 determined graphically from planimeter readings of V between surfaces 1 and 2 obtained from superimposed surface profile traces η also determined graphically $\lambda_{1,2} = (V/A_a)/(1 - k_f/k_{1,2})$; $k_f \neq k_{1,2}$	1) Contact and gap conductance 2) Single contact with air, water, and mercury in gaps 3) Followed by test of rough-milled Armco iron and aluminum contact
16 (semiempirical)	$$\frac{h_{c+g,S}\sigma}{km} = \frac{\sigma k_f}{km\lambda} + \frac{2\sigma}{m\psi(a_S/b_S)}\sqrt{\frac{P\sin\theta}{\pi\omega_1\omega_2 H_M}}$$ $\lambda = 4/9 \times (\sigma_{1,max} + \sigma_{2,max})$ $\psi(a_S/b_S) = 1$, typically ω_1, ω_2 = wavelengths transverse to grooves (=1 for data by Hegazy[27] in Figs. 2a and 2b; i.e., not grooved) θ = transverse slope of grooves ($=\pi/2$ for data by Hegazy[27] in Figs. 2a and 2b; i.e., not grooved)	1) Contact and gap conductance 2) Machined parallel grooves 3) Steel, brass, and aluminum 4) $\sigma = 4.3$–51 μm 5) air, water, and glycerol
17 (semiempirical)	$$\frac{h_{c+g,S}\sigma}{km} = \frac{\sigma P}{m[(\lambda_1+\lambda_2)/2 + \lambda_{oxide}]C_1 H_B} + \frac{2\sigma k_f}{km(\lambda_1+\lambda_2)}$$ λ_1, λ_2 not specified, probably $\approx \sigma_1$, σ_2, respectively λ_{oxide} = small length correction for oxide on surfaces $0 < C_1 \leq 1.0$, elasto–plastic deformation correction factor $C_1 = 1.0$, totally plastic	1) Contact and gap conductance 2) Gunmetal with air, oil, and paraffin
20 (semiempirical)	$h_{c,S}\sigma/km = 2.1 \times 10^4(\sigma/m)(P/3S_U)$	1) Assumed $a_S = 30$ μm, independent of σ and L 2) Steel, copper, aluminum, nickel, and uranium
21 (semiempirical)	$h_{c,S}\sigma/km = 8.0 \times 10^3(\sigma/m)(C_1P/3S_U)^{0.86}$ $C_1 = 1$ for $\sigma_{1,CLA} + \sigma_{2,CLA} \geq 30$ μm $C_1 = [30/(\sigma_{1,CLA} + \sigma_{2,CLA})]^{1/3}$ for 10 μm $\leq \sigma_{1,CLA} + \sigma_{2,CLA} < 30$ μm $C_1 = 15/(\sigma_{1,CLA} + \sigma_{2,CLA})$ for $\sigma_{1,CLA} + \sigma_{2,CLA} \leq 10$ μm	1) Assumed $a_S = 40$ μm 2) Aluminum, uranium, iron, magnox, steel, duraluminum, and niobium 3) $\pm 20\%$ error 4) $C_1P/3S_U \leq 2.5 \times 10^{-2}$ 5) $T \leq T_{melt}$
18 (semiempirical)	$$\frac{h_{c+g,S}\sigma}{km} = \frac{\sigma k_f}{mk\lambda} + \frac{\sigma C_1\sqrt{P/H_M}}{m\lambda\tan^{-1}\left(\sqrt{\frac{1}{P/H_M} - \frac{k_f}{h_{c+g,S}\lambda P/H_M}} - 1\right)}$$ $\lambda = 3.56(\sigma_{1,CLA} + \sigma_{2,CLA})$ for $\sigma_{1,CLA} + \sigma_{2,CLA} < 7.5$ μm $\lambda = 0.528(\sigma_{1,CLA} + \sigma_{2,CLA})$ for $\sigma_{1,CLA} + \sigma_{2,CLA} \geq 7.5$ μm $C_1 = 0.355 \times (P/H_M)\cdot(1.46\lambda^{-0.18})$	1) Contact and gap conductance 2) λ in μin.
52 (semiempirical)	$h_{c,S}\sigma/km = 0.69(P/3S_Y)^{0.85}$	1) Nominally flat, rough surfaces
19 (empirical)	$h_{c,S+L}\sigma/km = (\sqrt{\pi}\sigma/m\lambda)[3.8 \times 10^{-6}(\lambda_{P=0}/b_L) + 0.026(P/E)BT]^{0.56}$ $\lambda_{P=0} = [TIR_{rgh} + 2\sigma_{rgh} - 0.5(TIR_{smth} + 2\sigma_{smth})]$ $\lambda = \lambda_{P=0} \times \exp(-180 \times PBTb/E\lambda_{P=0})$	1) 400 data for aluminum, stainless steel, brass, and magnesium 2) overall error 24% 3) $b_L = 12.7$ mm
22 (semiempirical)	$h_{c,S}\sigma/km = 2.95 \times 10^3(\sigma/m)(C_1P/3S_U)^{0.66}$ $C_1 = 1$ for $\sigma_{1,CLA} + \sigma_{2,CLA} \geq 30$ μm $C_1 = [30/(\sigma_{1,CLA} + \sigma_{2,CLA})]^{1/3}$ for 10 μm $\leq \sigma_{1,CLA} + \sigma_{2,CLA} < 30$ μm $C_1 = 15/(\sigma_{1,CLA} + \sigma_{2,CLA})$ for $\sigma_{1,CLA} + \sigma_{2,CLA} \leq 10$ μm	1) Assumed $a_S = 40$ μm 2) 92 data for turned, ground, and lapped stainless steel and molybdenum 3) $2 \times 10^{-4} \leq C_1P/3S_U \leq 8 \times 10^{-3}$
25 (empirical)	$h_{c,S}\sigma/km = C_1(\sigma^2/Am)(L/\sigma^2H)^{C_2}$ $C_1 = 9.5$, $C_2 = 0.74$; stainless steel; $C_1 = 1.9$, $C_2 = 0.72$; aluminum	1) 102 stainless steel data; 240 aluminum data 2) Nominally flat, rough surfaces

Table 1 Empirical and semiempirical correlations for thermal contact conductance of nominally flat (arbitrarily nonflat), rough metals (continued)

Reference	Correlation	Remarks
26 (empirical)	$h_{c,S}\sigma/km = 3.73(\sigma^2/Am)(L/\sigma^2 H)^{0.66}$	1) 344 aluminum data including the 240 data compiled by Thomas and Probert[25] 2) Nominally flat, rough surfaces
23 (semiempirical)	$h_{c,S}\sigma/km = 2.1 \times 10^4(\sigma/m)(P/3S_U)$	1) $a_S = 30$ μm, independent of σ and L 2) Nominally flat, rough surfaces 3) Identical to model of Shlykov and Ganin[20] 4) Valid for $P/3S_U << 1$
24 (semiempirical)	$h_{c,S}\sigma/km = 2.7 \times 10^4(\sigma/m)(C_1 P/3S_U)^{0.956}$ $C_1 = 12/(\sigma_{1,\max} + \sigma_{2,\max})$ for 1 μm $\le \sigma_{1,\max} + \sigma_{2,\max} < 5$ μm $C_1 = [20/(\sigma_{1,\max} + \sigma_{2,\max})]^{0.63}$ for 5 μm $\le \sigma_{1,\max} + \sigma_{2,\max} < 10$ μm $C_1 = [30/(\sigma_{1,\max} + \sigma_{2,\max})]^{0.4}$ for 10 μm $\le \sigma_{1,\max} + \sigma_{2,\max} \le 30$ μm	1) 80 data for variety of materials 2) Nominally flat, rough surfaces 3) Assumed $a_S = 30$ μm
53 (empirical)	$h_{c,S}\sigma/km = 12.29 \times 10^{-3}(1/m)(P/H)^{0.66}$	1) 78 data for Zircaloy-2/uranium dioxide interfaces
28 (semiempirical)	$h_{c,S}\sigma/km = 44.6(\sigma/m)^{0.25}(P/c_1)^{0.97}$	1) c_1 = microhardness correlation coefficient, N/m^2 2) Nominally flat, rough surfaces

This correlation is applicable only to those four alloys just listed. Hegazy[27] and Song and Yovanovich[28] improved the Yovanovich[29] correlation with the appropriate value of H_c.

DeVaal et al.[30] modified the correlation by Yovanovich[29] to allow for anisotropic (e.g., grooved) surfaces. They observed that perpendicular orientation of the grooves of the contacting surfaces resulted in greater conductance than parallel orientation. Yovanovich and Nho[31] observed that conductance was greater for heat flux from rougher (ground) surfaces to smoother (lapped) surface than vice versa.

McWaid[32] determined experimentally that measured roughness and profile slope decreased moderately (usually no more than 15%) with an increase in sampling interval size from 2.5 to 25 μm. Majumdar and Tien,[33] in a novel approach, modeled contact surfaces as fractal networks, rather than Gaussian distributions of asperity heights as do nearly all other theories.

Sridhar and Yovanovich[34] combined the plastic deformation model by Yovanovich[29] with the elastic deformation model by Mikic[35] to allow for elasto–plastic deformation.

D. Theoretical Models for Nonflat, Rough Metals

Clausing and Chao[3] were the first to consider nonflat surfaces from a theoretical standpoint. They developed a theory for determining the macrocontact (large-scale) and microcontact (small-scale) resistances, $R_{c,L}$ and $R_{c,S}$, respectively, for relatively smooth, spherical surfaces in contact (Table 3 and Fig. 1). They determined the macrocontact radius $a_{L,\mathrm{Hz}}$ from the Hertz[36] contact model (also available in Timoshenko and Goodier[37]) for elastic smooth (i.e., no roughness) spheres. Clausing and Chao[3] then used the ratio of $a_{L,\mathrm{Hz}}$ to specimen radius b_L in the Roess[10] constriction resistance model to determine $R_{c,L}$. They also derived $R_{c,S}$, assuming the radii of the microcontacts (all of which are contained within $a_{L,\mathrm{Hz}}$) are independent of load and equal to the roughness. The portion of their model for calculating $R_{c,S}$ is based on plastic deformation.

The model by Clausing and Chao[3] is applicable when TIR or δ is several times greater (by a factor of 29 to 176 in their experiments) than the roughness σ, for which $R_{c,L}$ is many times greater than $R_{c,S}$. Their experiments for smooth stainless steel, brass, magnesium, and rougher aluminum agreed well with their theory.

Clausing and Chao[3] ignored the substantial redistribution of load farther from the center of contact that is caused when roughness and flatness deviation are of the same order. This load redistribution reduces $R_{c,L}$ by enlarging a_L, with respect to $a_{L,\mathrm{Hz}}$. Kitscha and Yovanovich[38] and Fisher and Yovanovich[39] experimentally verified the model by Clausing and Chao[3] for smooth, spherical carbon steel, tool steel, and nickel specimens with very small radii of curvature so that a_L typically equaled 2 mm or less.

All of the previously discussed theories, except the theory of Clausing and Chao,[3] assume the macrocontact pressure distribution is uniform over the entire apparent contact area, A_{app}. Clausing and Chao[3] assumed the contact pressure to be of uniform intensity within the Hertz[36] macrocontact region ($0 \le r \le a_{L,\mathrm{Hz}}$) and zero outside this region ($a_{L,\mathrm{Hz}} < r \le b_L$). In fact, the pressure over the macrocontact region is immaterial to their calculation of $R_{c,L}$, since they assumed perfect contact inside the Hertz[36] macrocontact region ($0 \le r \le a_{L,\mathrm{Hz}}$) to apply the Roess[10] constriction equation. To calculate $R_{c,S}$, they assumed the macrocontact pressure to be uniform over the Hertz[36] macrocontact region. That is, they computed pressure with respect to the area of the Hertz[36] macrocontact region ($P = L/A_{\mathrm{Hz}} = L/\pi a^2_{L,\mathrm{Hz}}$), instead of the apparent area of the surface ($P = L/A_{\mathrm{app}} = L/\pi b^2_L$). In reality, according to Hertz,[36] for spherical surfaces the macrocontact pressure exhibits a hemispherical distribution, decreasing continuously from a maximum value, $P_{0,\mathrm{Hz}}$, at the center of contact ($r = 0$) to zero at the boundary of the Hertz[36] macrocontact region ($r = a_{L,\mathrm{Hz}}$).

Mikic and Rohsenow[40] derived theoretical models for the contact conductance of cylinders and spheres. For spherical surfaces they determined $R_{c,L}$ from the Roess[10] constriction resistance model. They accounted for the increase of a_L beyond $a_{L,\mathrm{Hz}}$, due to roughness σ. Mikic and Rohsenow[40] verified their model against the three experiments for slightly spherical surfaces with TIR approximately equal to σ. Their expressions (Table 3) are tediously evaluated.

Greenwood and Tripp[41] performed the first in-depth analytical study of the effect of roughness on the pressure distribution and deformation of contacting, elastic spheres. They developed an iterative numerical model for predicting the pressure distribution and deformation and presented their results in dimensionless form (Table 4). The most important trends in their results are that increases in roughness decrease the maximum contact pressure P_0 (always at the center of contact), relative to the Hertz[36] maximum pressure $P_{0,\mathrm{Hz}}$, and extend a_L beyond $a_{L,\mathrm{Hz}}$. For a given set of conditions (i.e., load, geometry, and elastic constants), the solution by Greenwood and Tripp[41] yields a wide range of numerical solutions, depending on the prescribed mean plane separation u_0. Only one solution is physically possible, and so the numerical model by Greenwood and Tripp[41] is underconstrained.

Table 2 Theoretical models for thermal contact conductance of flat (optically flat or nearly optically flat), rough metals

Reference	Correlation	Remarks
1 (elastic)	$\frac{h_{c,S}\sigma}{km} = \sqrt{2}\,\frac{\sigma}{m}\,\frac{\eta\sqrt{\mathrm{erfc}(u^*/\sqrt{2})\sigma\beta I_1(u^*)}}{[1-\sqrt{\eta\pi\sigma\beta I_1(u^*)}]^{1.5}}$ $h_{c,S}\sigma/km = C_1(\sqrt{2}P/E'm)^{0.95}$ $C_1 = 1.87$, $\alpha = 15$; $C_1 = 1.75$, $\alpha = 5$	1) Flat, rough surfaces
54 (elastic)	$\frac{h_{c,S}\sigma}{km} = \frac{2}{5\Delta x}\,\frac{\sigma}{m}\,\frac{\sqrt{I_{0,0}(u^*)I_{1,1}(u^*)}}{[1-\sqrt{0.2\pi I_{1,1}(u^*)}]^{1.5}}$ $h_{c,S}\sigma/km = C_1(\sqrt{2}P/E'm)^{0.97}$ $C_1 = 2.80$, $\alpha = 60$; $C_1 = 2.38$, $\alpha = 4$	1) Flat, rough surfaces 2) Recasting of Whitehouse and Archard[2]
35 (elastic)	$\frac{h_{c,S}\sigma}{km} = \frac{1}{4\sqrt{\pi}}\,\frac{\exp(-u^{*2}/2)}{[1-\sqrt{0.25\,\mathrm{erfc}(u^*/\sqrt{2})}]^{1.5}}$ $h_{c,S}\sigma/km = 1.55(\sqrt{2}P/E'm)^{0.94}$	1) Flat, rough surfaces 2) Based on Cooper et al.[12] 3) $A_{\mathrm{real,el}} = 1/2 \times A_{\mathrm{real,pl}}$
55 (elastic)	$\frac{h_{c,S}\sigma}{km} = \frac{1}{(2\pi)^{5/4}}\,\frac{\exp(-u^{*2}/2)/\sqrt{u^*}}{\left(1-\sqrt{\frac{\exp(-u^{*2}/2)}{2\sqrt{2\pi}u^*}}\right)^{1.5}}$ $h_{c,S}\sigma/km = 0.799(\sqrt{2}P/E'm)^{0.98}$	1) Flat, rough surfaces 2) Valid for $u^* \geq 2.0$
56 (elastic)	$h_{c,S}\sigma/km = (2C_2/mC_1)(\sigma/\beta)^{(1-C_3)/2}(P/E)^{C_3}$ $0.05 \leq C_1 \leq 0.1$; $3.63 \leq 2C_2 \leq 4.85$; $0.93 \leq C_3 \leq 0.95$	1) Flat, rough surfaces 2) Recast by Sayles and Thomas[57]
33 (elastic and plastic)	$\frac{h_{c,S}\sigma}{km} = \frac{1}{m}\,\frac{1}{C_2(D, A_r/A_a)\sqrt{4-2D}}\left(\frac{(2-D)A_r/A_a}{D}\right)^{D/2}$ $A_{\mathrm{real}}/A_{\mathrm{app}} = (P/H)^{C1}$ $D = 1.0$, $C_1 = 1$, plastic; $D = 1.5$, $C_1 = 1.33$, elastic; $D = 2.0$, $C_1 = 1$, plastic	1) $C_2(D, A_{\mathrm{real}}/A_{\mathrm{app}})$ given by Majumdar and Bhushan[58]
40 (plastic)	$h_{c,S}\sigma/mk = 0.9(P/H_V)^{0.941}$	1) Flat, rough surfaces
1 (plastic)	$\frac{h_{c,S}\sigma}{km} = 2\,\frac{\sigma}{m}\,\frac{\eta\sqrt{\mathrm{erfc}(u^*/\sqrt{2})\sigma\beta I_1(u^*)}}{[1-\sqrt{\eta 2\pi\sigma\beta I_1(u^*)}]^{1.5}}$ $h_{c,S}\sigma/km = C_1(P/H_V)^{0.98}$ $C_1 = 1.91$, $\alpha = 15$; $C_1 = 1.51$, $\alpha = 5$	1) Flat, rough surfaces
12 (plastic)	$\frac{h_{c,S}\sigma}{km} = \frac{1}{2\sqrt{2\pi}}\,\frac{\exp(-u^{*2}/2)}{[1-\sqrt{0.5\,\mathrm{erfc}(u^*/\sqrt{2})}]^{1.5}}$ $h_{c,S}\sigma/km = 1.45(P/H_c)^{0.985}$ $3.6 \times 10^{-4} \leq P/H \leq 1.0 \times 10^{-2}$ $1.0\ \mu\mathrm{m} \leq \sigma \leq 8.0\ \mu\mathrm{m}$; $0.08 \leq m \leq 0.16$	1) Nominally flat, rough surfaces for light to moderate relative pressure
29 (plastic)	$\frac{h_{c,S}\sigma}{km} = \frac{1}{2\sqrt{2\pi}}\,\frac{\exp(-u^{*2}/2)}{[1-\sqrt{0.5\,\mathrm{erfc}(u^*/\sqrt{2})}]^{1.5}}$ $h_{c,S}\sigma/km = 1.25(P/H_c)^{0.95}$	1) Nominally flat, rough surfaces 2) Recorrelation of same model as Cooper et al.[12]

Tsukada and Anno[42] borrowed from the work of Greenwood and Tripp[41] and developed a similar expression (Table 4) for the pressure distribution between rough, metallic spheres. They experimentally verified the increase in a_L, due to roughness. They introduced two dimensionless parameters, $P_0/P_{0,\mathrm{Hz}}$ and $a_L/a_{L,\mathrm{Hz}}$, relating, respectively, the actual to the Hertz[36] maximum pressure and the effective to the Hertz[36] macrocontact radius. They, as well as Sasajima and Tsukada[43] in a related study, presented these two dimensionless parameters in graphical form for discrete cases. However, they did not unify their results into a general relationship from which $P_0/P_{0,\mathrm{Hz}}$ and $a_L/a_{L,\mathrm{Hz}}$ could be predicted, relying instead on experimental determination of these parameters for each particular contact.

To account for a nonuniform pressure distribution, Mikic[44] developed expressions (Table 3) for the $R_{c,L}$ and $R_{c,S}$ for surfaces with one-dimensional variations in contact pressure [i.e., $P(r)$ and $P(x)$]. Mikic[44] did not address how to determine the pressure distribution.

Roca and Mikic[45] examined the contact conductance of bolted joints (Table 3). They predicted the increase in the effective contact radius due to roughness. Although $R_{c,S}$ increases with increasing roughness, Roca and Mikic[45] reasoned that $R_{c,L}$ would decrease due to the larger a_L. The net effect can be either an increase or decrease in total resistance.

Thomas and Sayles[46] investigated the relationship between roughness, flatness deviation, and contact resistance. They noted that TIR is useful only if it is assumed that the surface is spherical. They also observed that the theories for flat, rough surfaces in Table 2, published prior to 1974 (as well as those published since then), all predict the slope of conductance vs load to be between 0.94–0.99, whereas the slope predicted from the theory by Hertz[36] for smooth spheres is 0.333. The correlations of Mal'kov,[22] Fletcher and Gyorog,[19] Thomas and Probert,[25] and O'Callaghan and Probert,[26] all developed for nominally flat surfaces, predict slopes between 0.56–0.74. Thomas and Sayles[46] speculated that these intermediate slopes

Table 3 Theoretical models for thermal contact conductance of spherical (nonflat), rough metals

Reference	Correlation	Remarks
3	$\frac{h_{c,L}b_L}{k} = \frac{2}{\pi}\frac{a_{L,Hz}/b_L}{\psi(a_{L,Hz}/b_L)} = \frac{2\times 1.285\chi^{1/3}}{\pi\psi(1.285\chi^{1/3})}$ $\frac{h_{c,S}\sigma}{km} = \frac{\sigma}{m}\frac{2}{\pi}\frac{L/\pi a_{L,Hz}^2}{C_1 H}\frac{1}{a_S\psi(a_S/b_S)}$ $\frac{R_{c,L}}{R_{c,S}} = \frac{L/\pi a_{L,Hz}^2}{C_1 H}\frac{\psi(1.285\chi^{1/3})}{1.285\chi^{1/3}}\frac{b_L}{a_S\psi(a_S/b_S)}$ $a_{L,Hz} = (0.75L\rho/E')^{1/3}$ $0 \le C_1 \le 1.0$; $C_1 = 1.0$ for plastic deformation $\chi = [(L/\pi b_L^2)/E_{harm}][b_L/(\delta_1 + \delta_2)]$ R_L/R_S (maximum) = 176 for smooth magnesium R_L/R_S (minimum) = 29 for rough aluminum	1) Spherical, relatively smooth surfaces 2) Elastic macroscopic deformation, plastic microscopic deformation 3) Assumed $a_S = \sigma$ 4) Stainless steel, brass, aluminum, magnesium 5) C_1 = elasto–plastic deformation correction factor 6) b_L = 12.7 mm
40	$\frac{h_{c,L}b_L}{k} = \frac{2}{\pi}\frac{a_L/b_L}{\psi(a_L/b_L)} = \frac{2}{\pi}\frac{\varepsilon_L}{\psi(\varepsilon_L)} = \frac{2\times 1.285\chi^{1/3}}{\pi\psi(1.285\chi^{1/3})}$ $\varepsilon_L^2 = \varepsilon_{L,Hz}^2 + 2\int_{\varepsilon_{L,Hz}}^{1} \exp\left[-\frac{\delta}{\sigma}\varepsilon_{L,Hz}^2 C_1\left(2u^* + \frac{\delta}{\sigma}\varepsilon_{L,Hz}^2 C_1\right)\right]\varepsilon_L\, d\varepsilon_L$ $C_1 = \frac{\varepsilon_L^2}{\varepsilon_{L,Hz}^2} - 2\left\{1 - \frac{1}{\pi}\left[\left(2 - \frac{\varepsilon_L^2}{\varepsilon_{L,Hz}^2}\right)\sin^{-1}\left(\frac{\varepsilon_{L,Hz}}{\varepsilon_L}\right) + \sqrt{\frac{\varepsilon_L^2}{\varepsilon_{L,Hz}^2} - 1}\right]\right\}$ $h_{c,S}\sigma/km = 0.9(P/H)^{0.941}$	1) Spherical, rough surfaces 2) Elastic macroscopic deformation, plastic microscopic deformation 3) u^* and ε_L coupled, requires iterative solution for a_L
44	For axisymmetric pressure distribution $P(r)$: $\frac{h_{c,S}\sigma}{km} = 2.90\int_0^1 \frac{r}{b_L}\left(\frac{P}{H}\right)^{0.985} d\left(\frac{r}{b_L}\right)$ $\frac{h_{c,L}b_L}{k} = \frac{1}{8}\left\{\sum_{n=1}^{\infty}\frac{\left[\int_0^1 \frac{r}{b_L}\left(\frac{P}{P_{av}}\right)^{0.985} J_0\left(\zeta_n\frac{r}{b_L}\right) d\left(\frac{r}{b_L}\right)\right]^2}{\zeta_n J_0^2(\zeta_n)}\right\}^{-1}$ For unidirectional pressure distribution $P(x)$: $\frac{h_{c,S}\sigma}{km} = 1.45\int_0^{\infty}\left(\frac{P}{H}\right)^{0.985} d\left(\frac{x}{b_L}\right)$ $\frac{h_{c,L}b_L}{k} = \frac{1}{4}\left\{\sum_{n=1}^{\infty}\frac{1}{n}\left[\int_0^{\infty}\left(\frac{P}{P_{av}}\right)^{0.985}\cos\left(\frac{n\pi x}{b_L}\right) d\left(\frac{x}{b_L}\right)\right]^2\right\}^{-1}$	1) h_S and h_L for $P = P(r)$ applicable to spherical surfaces; h_S and h_L for $P = P(x)$ applicable to cylindrical surfaces 2) Microscopic conductance employs plastic deformation model of Cooper et al.[12] 3) $P(r)$ and $P(x)$ undefined
45	$\frac{P(r)}{P_O} = \frac{H_V}{2P_O}\,\text{erfc}\left[\frac{\frac{w(r)E}{t_{plate}P_O} + \frac{w(0)E}{t_{plate}P_O}}{\frac{\sqrt{2}\sigma E}{t_{plate}P_O}}\right]$ $\frac{h_{c,L}(r)t_{plate}}{k} = \frac{1.45Em}{H_V}\frac{P(r)t_{plate}}{\sigma E}$	1) Bolted plates 2) Rough, perfectly flat before loading 3) $P(r)$ and $w(r)$ solved iteratively
47	$H^* = \pi H[\frac{9}{16}(\rho^2/E'^2L)]^{1/3}$ $\sigma^* = \pi\sigma\,[\frac{3}{4}[\rho E'^2/L^2)]^{1/3}$ $m^* = mE'/H$ $\frac{h_{c,L}b_L}{k} = \frac{\frac{b_L}{a_{L,Hz}}\frac{\pi^2}{4}\left(\sum_{i=1}^{N} C_{1,i}^2\right)^2}{\sum_{i=1}^{N} C_{1,i}\left(\frac{4}{3}C_{1,i}^2 + \sum_{j\neq i}^{N} C_{1,j}^2\sum_{k=1}^{n} C_{2,k}C_{3,ij}\right)}$ $C_{1,i\,\text{or}\,j} = \frac{a_{S,i\,\text{or}\,j}}{a_{L,Hz,i\,\text{or}\,j}}$ $C_{2,k}$ = weight factor $C_{3,ij} = \sin^{-1}C_{4,ij}$ isothermal $C_{3,ij} = C_{4,ij}K(C_{4,ij})$ uniform heat flux $C_{4,ij} = C_{1,i}\Big/\left[\left(\frac{r_j\cos\theta_j}{a_{L,Hz}} + \frac{x_k}{a_{L,Hz}} - \frac{r_i\cos\theta_i}{a_{L,Hz}}\right)^2 + \left(\frac{r_j\sin\theta_j}{a_{L,Hz}} + \frac{y_k}{a_{L,Hz}} - \frac{r_i\sin\theta_i}{a_{L,Hz}}\right)^2\right]$	1) Spherical, rough surfaces 2) Elastic macroscopic deformation, plastic microscopic deformation 3) Verified only for steel sphere: ρ = 14.29 mm, σ = 1.3 μm, and $a_L \le$ 0.5 mm
49	$P(r)$ defined by Tsukada and Anno[42]: $R_{c,S}[P(r)] = 1/h_{c,S}[P(r)]$ and $R_{c,L}[P(r)] = 1/h_{c,L}[P(r)]$ defined by Mikic[44]: $h_{c,S+L} = 1/(R_{c,S} + R_{c,L})$	1) Spherical, rough surfaces 2) Elastic macroscopic deformation, plastic microscopic deformation 3) $P_O/P_{O,Hz}$ and $a_L/a_{L,Hz}$ determined from experiments with pressure-sensitive films

for a great many experimental data are due to the importance of both roughness and TIR in most practical situations. They go on to assert, that for high loads on small smooth surfaces of low elastic modulus, the surfaces will flatten and $R_{c,S}$ will dominate, whereas for low loads on large rough surfaces of high elastic modulus, the surfaces will remain spherical and $R_{c,L}$ will dominate.

Burde and Yovanovich[47] also considered the contact resistance of rough spheres. Their model (Table 3) demonstrated good agreement with experimental data for rough, spherical specimens. However, they tested only specimens with small radii of curvature (ρ = 14.29 mm) that yielded values of a_L of only 0.5 mm or less.

Schankula et al.[48] studied the cylindrical contact resistance between concentric tubes. They modeled the nonideal tube surfaces as sinusoidal waves. They reported significant disagreement between their theory and experimental data.

Nishino et al.[49] recast the contact pressure model by Tsukada and Anno[42] for rough, spherical metals (Table 3) and combined it with the thermal model introduced by Mikic.[44] Nishino et al.[49] verified their thermomechanical model in four experiments involving rough, spherical specimens. Nishino et al.[49] used pressure-sensitive films for determining the dimensionless pressure distribution parameters, $P_0/P_{0,Hz}$ and $a_L/a_{L,Hz}$. However, Nishino et al.[49] did not present general relationships between pressure distribution and surface profile that would eliminate the need for pressure measurements.

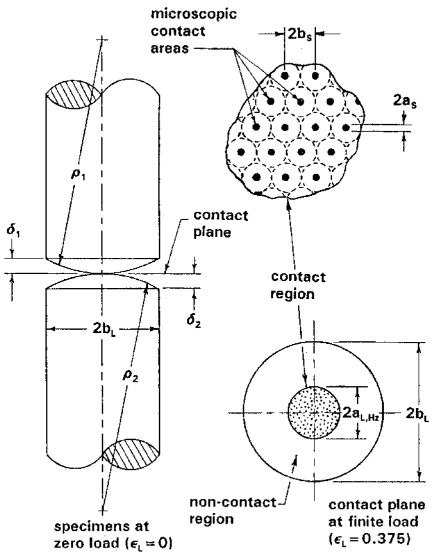

Fig. 1 Contacting spherical surfaces considered by Clausing and Chao.[3]

II. Comparisons of Models

A. Theories for Conforming (Flat), Rough Metals

Sridhar and Yovanovich[50] critiqued most of the previously listed (Table 2) elastic and plastic deformation theories for conforming, rough metallic surfaces. They compared these several theories to many carefully controlled experiments with nearly optically flat (TIR $\leq$ 2.0 μm) specimens of stainless steel 304, nickel 200, zirconium-2.5% niobium, and Zircaloy-4 alloys performed by Antonetti,[51] Hegazy,[27] and McWaid.[32] Sridhar and Yovanovich[50] expressed the theoretical correlations in terms of three dimensionless parameters to facilitate comparisons: 1) $h\sigma/km$, dimensionless conductance; 2) $2^{1/2}P/E'm$, dimensionless pressure for elastic deformation; and 3) P/H_c, dimensionless pressure for plastic deformation. Sridhar and Yovanovich[50] demonstrated (Figs. 2a and 2b) that, of all the theories they examined, the correlation of Yovanovich[29] most closely agrees with $R_{c,S}$ data for these four alloys.

B. Empirical Correlations for Nominally Flat, Rough Metals

Hegazy[27] compared the model by Yovanovich,[29] which performs better than all other theories in Table 2 for flat, rough metals, to many (nine) of the empirical and semiempirical correlations in Table 1. Hegazy[27] based his comparison on two extreme cases of his meticulously obtained experimental data: a very rough pair of specimens of a low conductivity material (σ = 10.95 μm, stainless steel 304: k = 16.2 W/mK), and a fairly smooth pair of specimens of a moderately high conductivity material (σ = 0.902 μm, nickel 200: k = 70 W/mK). In the present study, an additional three correlations from Table 1 (12 total) are included in this comparison.

The remaining four correlations in Table 1 were omitted from the comparison. The correlation by Fenech and Rohsenow[14,15] requires actual surface traces of the contact surfaces.

Table 4 Contact models for rough spheres

Reference	Correlation	Remarks
41	$u^*(r^*) = u(r)/\sigma \quad u_0^* = u_0/\sigma \quad w^*(r^*) = w(r)/\sigma \quad w_0^* = w_0/\sigma$ $r^* = \frac{r}{\sqrt{2\rho\sigma}} \quad P^* = \frac{P}{E'\sqrt{\sigma/8\rho}} \quad L^* = \frac{2L}{\sigma E'\sqrt{2\rho\sigma}}$ $u^*(r^*) = u_0^* + r^{*2} + w^*(r^*) - w_0^*$ $P^*(r^*) = \left(\frac{8}{3}\eta\sigma\sqrt{2\rho\beta}\right)\int_{u^*}^{\infty}(z^* - u^*)^{3/2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^{*2}}{2}\right)\mathrm{d}z^*$ $w^*(r^*) = \int_0^{\infty} r^*P^*(r^*\xi)\kappa(\xi)\,\mathrm{d}\xi \quad (r^* > 0)$ $w_0^* = \int_0^{\infty} P^*(\xi)\,\mathrm{d}\xi \quad (r^* = 0)$ $\kappa(\xi) = (2/\pi)K(\xi) \quad \xi < 1 \quad \text{and} \quad \kappa(\xi) = (2/\pi)K(1/\xi) \quad \xi > 1$	1) Spherical, rough surfaces 2) Elastic macroscopic and microscopic deformation 3) Iterative solution for $P^*(r^*)$ and $w^*(r^*)$ 4) K is elliptic integral
42	$P(r) = \frac{P_0}{P_{0,Hz}}\frac{1}{\pi}\left(\frac{3LE_1^2}{2(1-\nu_1^2)^2\rho^2}\right)^{1/3}\left(1-\left\{\frac{r}{\frac{a_L}{a_{L,\mathrm{Hz}}}\left[\frac{3(1-\nu_1^2)L\rho}{2E_1}\right]^{1/3}}\right\}^2\right)^{\gamma}$ $\gamma = \frac{3}{2}\frac{P_0}{P_{0,Hz}}\left(\frac{a_L}{a_{L,Hz}}\right)^2 - 1$	1) Spherical, rough surfaces 2) Elastic macroscopic deformation, plastic microscopic deformation 3) Spherical, rough surfaces 4) $P_0/P_{0,Hz}$ and $a_L/a_{L,Hz}$ determined from heat-tinted steel specimens 5) Expression shown recast by Nishino et al.[49]

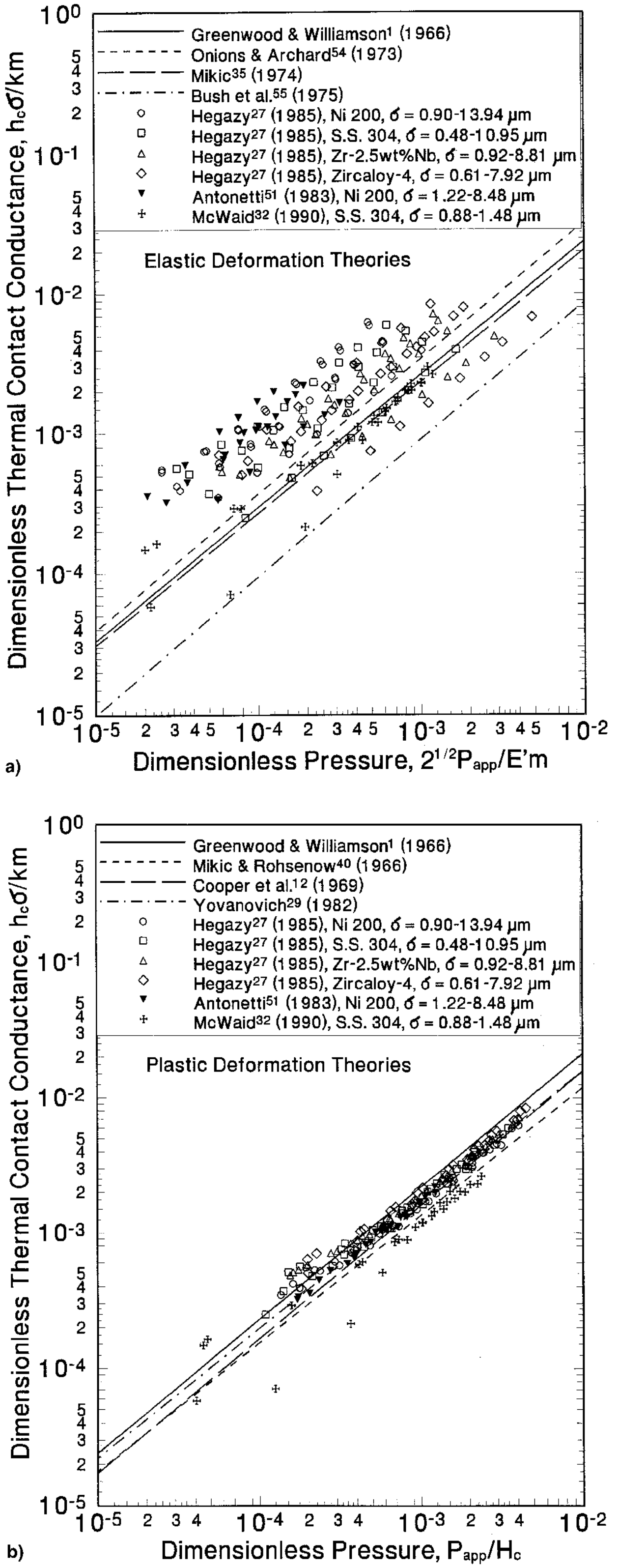

Fig. 2 a) Elastic and b) plastic deformation theories from Table 2 for flat, rough surfaces compared to experiments for four optically flat (TIR ≤ 2 μm) rough alloys.

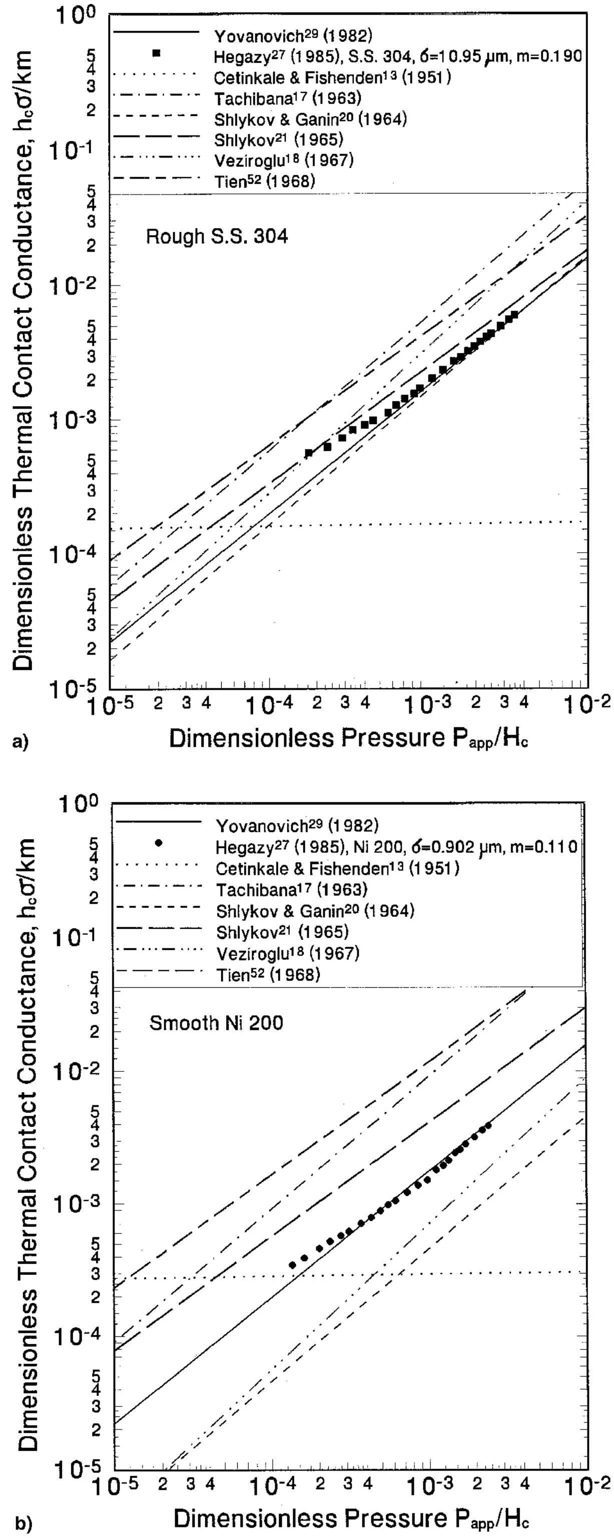

Fig. 3 Comparison of empirical correlations from Table 1, published before 1970, to the theoretical correlation by Yovanovich[29] and the a) lowest and b) highest experimental conductance (for rough stainless steel) by Hegazy.[27]

The model by Laming[16] is only applicable to directionally grooved surfaces, The correlation by Bochorishvili and Ganin[23] is identical to that by Shlykov and Ganin.[20] The model by Song and Yovanovich[28] is essentially the same as that by Hegazy[27] for the determination of H_c.

The various empirical and semiempirical correlations are compared to the theoretical model by Yovanovich[29] and the experimental data for the two extreme cases of rough stainless steel 304 and smooth nickel 200 by Hegazy[27] in Figs. 3a and 3b and 4a and 4b. The correlations were separated into two groups: those published before 1970 (Figs. 3a and 3b) and those published during or after 1970 (Figs. 4a and 4b) to avoid cluttering the figures.

The correlations that include gap conductance h_g (which is negligible for high vacuum conditions assumed in the present

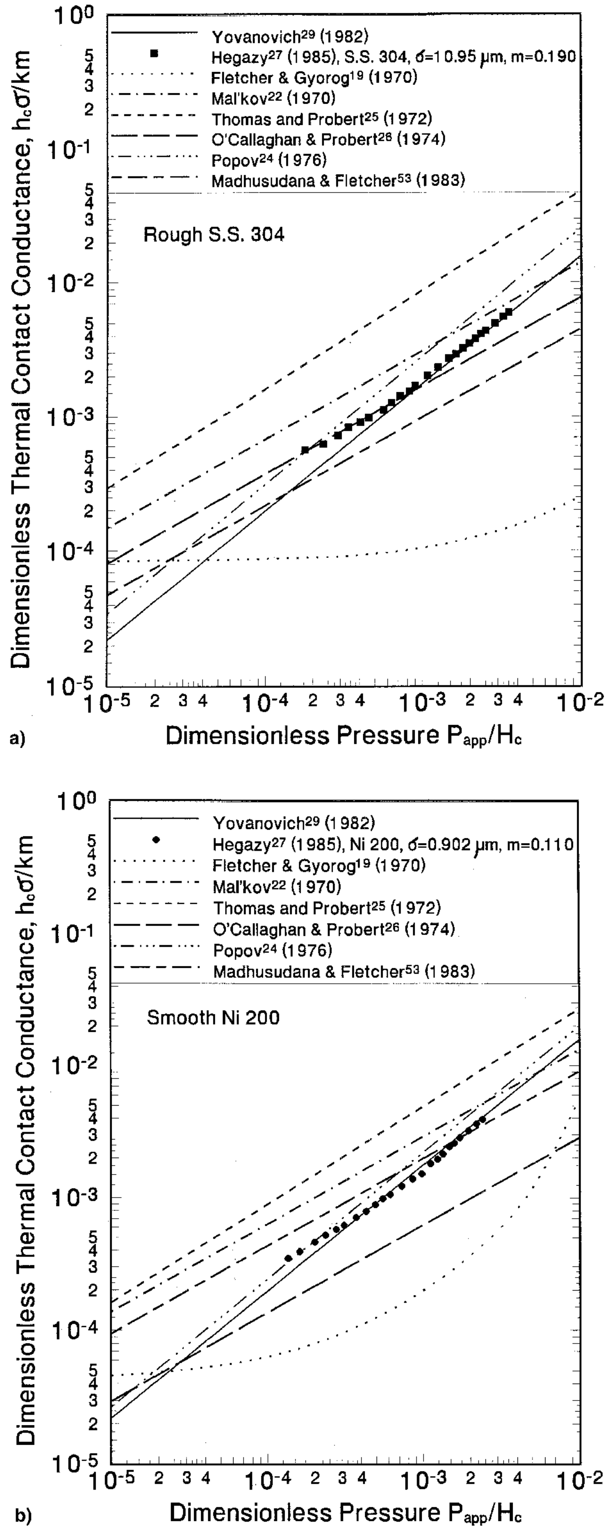

Fig. 4 Comparison of empirical correlations from Table 1, published during or after 1970, to the theoretical correlation by Yovanovich[29] and the a) lowest and b) highest experimental conductance (for rough stainless steel) by Hegazy.[27]

comparison), in addition to contact conductance h_c, do not compare well, in general, to the correlation by the Yovanovich[29] model and experimental data by Hegazy,[27] as shown in Figs. 3a and 3b. The conductance predicted by Cetinkale and Fishenden[13] is nearly independent of pressure and considerably lower than the experimental data by Hegazy,[27] suggesting that h_g was dominant. Conversely, the correlation by Tachibana[17] is markedly greater than the correlation by Yovanovich[29] and data by Hegazy.[27] The correlation by Veziroglu[18] approaches the data by Hegazy[27] only for rough stainless steel 304 at low contact pressure.

Most of the Russian correlations[20–22] approximate the correlation by Yovanovich[29] and experimental data by Hegazy[27] only for very rough surfaces that are relatively unworked. Unworked surfaces are not strain hardened to any great depth due to machining processes. In other words, the Russian investigators assumed H_c is approximately equal to H_B ($\approx 3S_U$). In light of the fact that Hegazy[27] demonstrated that H_c is typically significantly greater than H_B, the Russian correlations usually exhibit marked error, except for values of σ/m large enough to reduce H_c to approximately equal H_B. Consequently, the assumption by Tien,[52] that H_c equals three times the yield strength S_Y, results in even greater disagreement with the correlation by Yovanovich,[29] since S_Y is almost always less than S_U. The correlation by Popov[24] (Figs. 4a and 4b) performs rather well for both smooth nickel 200 and rough stainless steel 304.

The empirical correlations by Thomas and Probert[25] are different for stainless steel (with mechanical properties similar to nickel, thus allowing comparison) and aluminum. They suggested that this may due to some intrinsic property of the material for which they did not account. Hegazy[27] points out that, since the slopes of the two correlations by Thomas and Probert[25] are nearly equal, the missing variable is dimensionless and may be a surface parameter. Their correlation (Figs. 4a and 4b) is markedly greater than the correlation by Yovanovich[29] and the experimental data by Hegazy[27] for both cases.

The empirical correlation by O'Callaghan and Probert[26] (Figs. 4a and 4b), which is almost identical to the correlations by Thomas and Probert,[25] agrees somewhat with the correlation by Yovanovich[29] only for rough surfaces, but only at low contact pressure. This is contrary to the fact that the correlation by O'Callaghan and Probert[26] was developed for smooth surfaces.

The correlations by Thomas and Probert,[25] O'Callaghan and Probert,[26] Fletcher and Gyorog,[19] Madhusudana and Fletcher,[53] and Mal'kov[22] have substantially lower exponents (slopes) than the correlation by Yovanovich.[29] These low-valued exponents may be due to nonflatness.

The correlation by Yovanovich[29] agrees more closely with the experimental data by Hegazy[27] than do all the empirical and semiempirical correlations considered in the present study. However, the correlation by Popov[24] approximates the data by Hegazy[27] reasonably well.

C. Theories for Spherical, Rough Metals

The theory by Nishino et al.[49] for spherical, rough metals and their experimental results for four contacts are plotted in Figs. 5a–5d and compared to various other models. The theory by Nishino et al.[49] was chosen over other theories in Table 3 as a basis for comparison because it is presented in the most readily interpreted format and has the most associated experimental results. The correlation by Yovanovich[29] is plotted because his is the most accurate of the theoretical models for flat, rough surfaces in Table 2. The correlation by Yovanovich[29] is greater than the experimental data by a factor of 3 to 70 (Figs. 5a–5d). This factor increases with increasing ratio of flatness deviation to roughness, TIR/σ. This is expected since the correlation by Yovanovich[29] cannot account for $R_{c,L}$, which increases relative to $R_{c,S}$ for increasing TIR/σ.

Nishino et al.[49] also made comparisons with the empirical correlations of Thomas and Probert[25] and Fletcher and Gyorog,[19] as shown in Figs. 5a–5d. The latter two empirical correlations are based on some of the largest sets of data (340 and approximately 400, respectively) compiled to date. Hence, the expressions by Fletcher and Gyorog[19] and Thomas and Probert[25] are potentially the most successful empirical correlations available, bearing in mind the often inherently limited applicability of empirical correlations. The correlation by

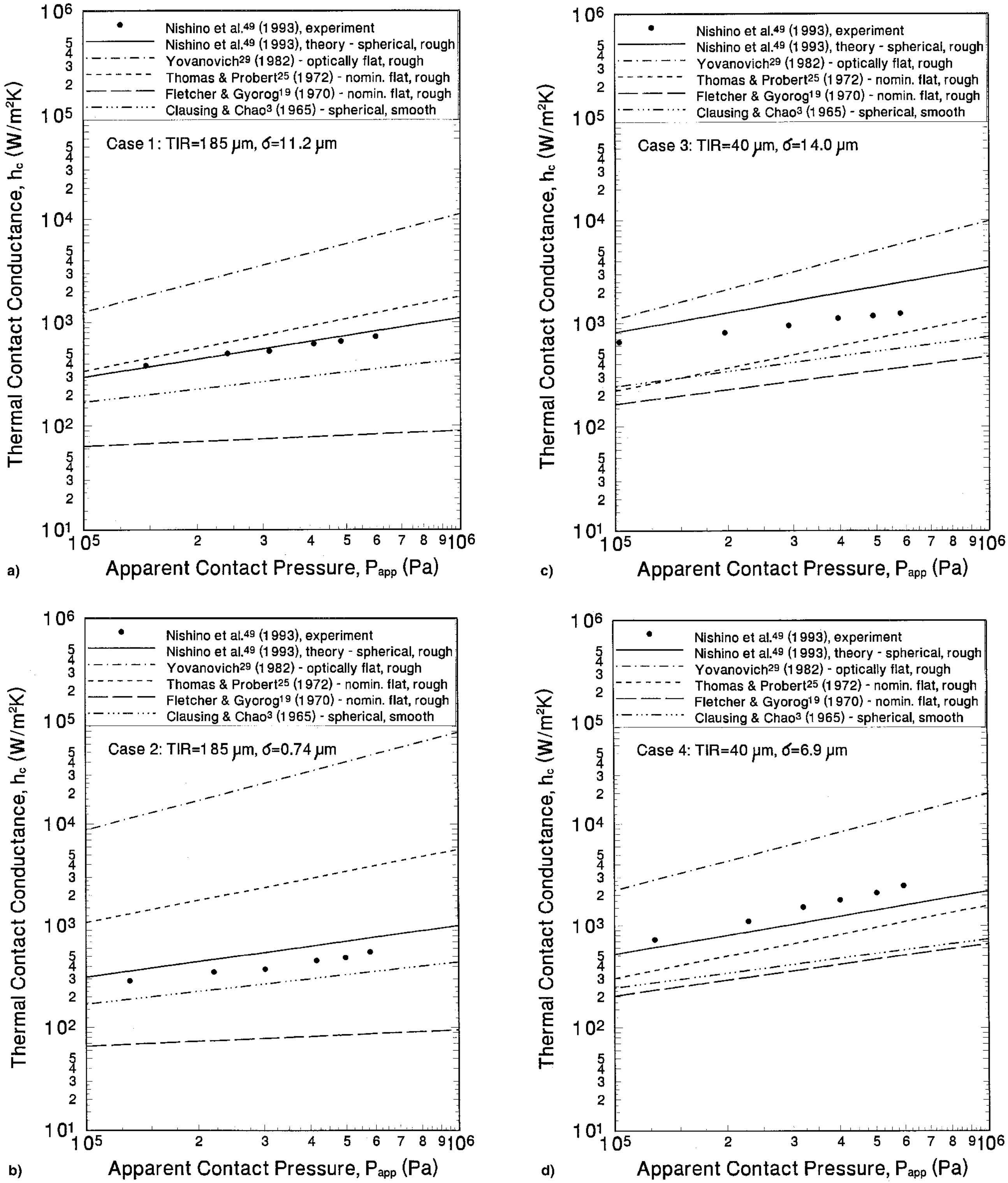

Fig. 5 Predicted and experimental thermal contact conductance vs apparent contact pressure for spherical, rough aluminum alloy from Nishino et al.[49] compared to various models. Cases a) 1, b) 2, c) 3, and d) 4.

Thomas and Probert[25] underpredicts the conductance of the two very rough, relatively flat specimen pairs (case 3: TIR/σ = 2.86 and case 4: TIR/σ = 5.80) by a factor of 2, as illustrated in Figs. 5c and 5d. Their correlation overpredicts the conductance of the markedly nonflat, smooth specimen pair (case 2: TIR/σ = 250) by a factor of 6, as shown in Fig. 5b, and agrees reasonably well with the markedly nonflat, very rough specimen pair (case 1: TIR/σ = 16.5), also shown in Fig. 5a. This suggests that the empirical correlation by Thomas and Probert[25] was obtained for significantly nonflat, very rough specimens, although it was reportedly obtained for smooth surfaces. The correlation by Fletcher and Gyorog[19] is 5 to 9 times less than all four experiments, indicating that their correlation is based on markedly nonflat specimens.

The theory by Clausing and Chao[3] for smooth, spherical surfaces is included in Figs. 5a–5d, since it represents the limiting case opposite that of the correlation by Yovanovich[29] for flat, rough surfaces. The theory by Clausing and Chao[3] is 1.5 to 4 times less than experimental data by Nishino et al.[49] The disagreement between the theory by Clausing and Chao[3] and the experimental data is more pronounced for lesser values of TIR/σ (e.g., cases 3 and 4, Figs. 5c and 5d). $R_{c,L}$ is less than that predicted from the Hertz[36] contact model, because the macrocontact radius is larger than the Hertz[36] macrocontact radius (i.e., $a_L/a_{L,Hz} > 1$), due to the significant roughness (i.e., smaller TIR/σ). The reduction in $R_{c,L}$ more than offsets the presence of $R_{c,S}$ due to appreciable roughness. The model by Mikic and Rohsenow[40] yields results that may hardly be distinguished from predictions obtained from the theory by Clausing and Chao,[3] and is therefore omitted from Figs. 5a–5d.

III. Summary and Conclusions

Theories exist that accurately predict thermal contact conductance of metals for two bounding cases of geometrical profiles: 1) nearly optically flat (flatness deviation less than approximately 2 μm), rough surfaces (e.g., Cooper et al.,[12] Mikic,[35] and Yovanovich[29]), and 2) spherical, smooth surfaces (Clausing and Chao[3]). Only $R_{c,S}$ is significant for flat, rough surfaces; whereas only $R_{c,L}$ is significant for spherical, smooth surfaces.

Most metal surfaces fabricated for contact heat transfer applications are termed nominally flat and rough. However, though called nominally flat, their actual degree of nonflatness (TIR or δ, illustrated in Fig. 1) is usually many times greater, often by a factor of 10 or 100, than the limit (2 μm) for optical flatness. For such surfaces both $R_{c,S}$ and $R_{c,L}$ are usually substantial. A number of empirical and semiempirical correlations (refer to Table 1) have been developed for predicting the contact conductance of nominally flat, rough surfaces, but none of these are widely successful.

Mikic and Rohsenow,[40] Thomas and Sayles,[46] Burde and Yovanovich,[47] and Nishino et al.[49] examined the contact conductance of (arbitrarily) nonflat, rough surfaces from a theoretical standpoint. They all modeled nonflat surfaces as spherical to make analysis tractable. This is reasonable because machined surfaces are often approximately spherical with large radii of curvature, or, if not spherical, are convex (also referred to as crowned) with monotonic curvature. The model by Nishino et al.[49] performs well for spherical, rough surfaces, but it is computationally intensive and is not presented in a format practical for design studies.

Acknowledgments

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